

QCD plasma thermalization, collective flow and extraction of shear viscosity \ddagger

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Abstract. Fast thermalization and elliptic flow of QCD matter found at the Relativistic Heavy Ion Collider (RHIC) are understood as the consequence of perturbative QCD (pQCD) interactions within a (3+1) dimensional parton cascade. The main contributions stem from pQCD-inspired bremsstrahlung. We extract the shear viscosity to entropy ratio, which is between 0.08 and 0.15.

\ddagger Invited talk.

1. Introduction and motivation

The values of the elliptic flow parameter v_2 measured by the experiments at the Relativistic Heavy Ion Collider (RHIC) [1] are so large as predicted by calculations employing the ideal hydrodynamics [2]. This finding suggests at first that a fast local equilibration of quarks and gluons occurs at a very short time scale, ≤ 1 fm/c, as used in the hydrodynamical calculations. Coherent quantum effects like color instabilities [3] may play a role in isotropization of particle degrees of freedom at the very initial stage where the matter is super dense. However, more quantitative studies are needed to determine their significance on the true thermal equilibration as suggested for the expanding quark-gluon matter at RHIC.

The second message stemming from a comparison between measured v_2 and that predicted by calculations is that the locally thermalized state of matter created, the quark-gluon plasma (QGP), behaves as a nearly perfect fluid exhibiting strong “explosive” collective motion. Quarks and gluons should be strongly coupled [4]. The reason for it is still an open issue. Furthermore, recent investigations [5] show that the QGP created should indeed possess a small viscosity coefficient. New viscous hydrodynamical calculations [6] are worked out to extract the viscosity from comparisons to experimental data. Drawbacks in these calculations are the assumed ideal thermal initial conditions and the assumed Bjorken longitudinal expansion. The smallness of the viscosity is of great interest as a result of the recent debate about speculative “realizations” of super symmetric representations of Yang-Mills theories using the AdS/CFT conjecture [7]. However, before one may believe that an introduction of extra effective degrees of freedom is really needed, one should also pursue more conservative explanations for strongly coupled system, using well-established theories. In this talk, we demonstrate that the perturbative QCD (pQCD) can still explain a fast thermalization of the initially nonthermal gluon system, the large collective effects of QGP created at RHIC and the smallness of the shear viscosity to entropy ratio in a consistent manner by using a relativistic pQCD-based on-shell parton cascade Boltzmann approach of multiparton scatterings (BAMPS) [8, 9, 10, 11]. In principle, there is no need to invoke exotic black hole physics in higher dimensions and super symmetric Yang-Mills theories using the AdS/CFT conjecture to understand RHIC data.

2. Parton cascade BAMPS

BAMPS is a parton cascade, which solves the Boltzmann transport equation and can be applied to study, on a semi-classical level, the dynamics of gluon matter produced in heavy-ion collisions at RHIC energies. The structure of BAMPS is based on the stochastic interpretation of the transition rate [8, 12, 13, 14], which ensures full detailed balance for multiple scatterings. BAMPS subdivides space into small cell units where the operations for transitions are performed.

Gluon interactions included in BAMPS are elastic pQCD $gg \rightarrow gg$ scatterings as

well as pQCD-inspired bremsstrahlung $gg \leftrightarrow ggg$. The differential cross sections and the effective matrix elements are given by [15]

$$\begin{aligned} \frac{d\sigma^{gg \rightarrow gg}}{dq_{\perp}^2} &= \frac{9\pi\alpha_s^2}{(q_{\perp}^2 + m_D^2)^2}, \\ |\mathcal{M}_{gg \rightarrow ggg}|^2 &= \frac{9g^4}{2} \frac{s^2}{(\mathbf{q}_{\perp}^2 + m_D^2)^2} \frac{12g^2\mathbf{q}_{\perp}^2}{\mathbf{k}_{\perp}^2[(\mathbf{k}_{\perp} - \mathbf{q}_{\perp})^2 + m_D^2]} \Theta(k_{\perp}\Lambda_g - \cosh y) \end{aligned} \quad (1)$$

where $g^2 = 4\pi\alpha_s$. \mathbf{q}_{\perp} and \mathbf{k}_{\perp} denote the perpendicular component of the momentum transfer and of the radiated gluon momentum in the center-of-mass frame of the collision, respectively. y is the momentum rapidity of the radiated gluon in the center-of-mass frame, and Λ_g is the gluon total mean free path, which is calculated self-consistently [8]. The interactions of the massless gluons are screened by a Debye mass $m_D^2 = \pi d_G \alpha_s N_c \int d^3p / (2\pi)^3 \cdot f/p$ where $d_G = 16$ is the gluon degeneracy factor for $N_c = 3$. m_D is calculated locally using the gluon density function f obtained from the BAMPS simulation. The suppression of the bremsstrahlung due to the Landau-Pomeranchuk-Migdal (LPM) effect is taken into account within the Bethe-Heitler regime employing the step function in equation (2).

The initial gluon distributions are taken as an ensemble of minijets with transverse momenta greater than 1.4 GeV [9], produced via semihard nucleon-nucleon collisions. We use Glauber geometry with Woods-Saxon profile and assume independent binary nucleon-nucleon collisions. A formation time for initial minijets is also included [8, 9].

In the present simulations, the interactions of the gluons are terminated when the local energy density drops below 1 GeV fm^{-3} . This value is assumed to be the critical value for the occurrence of hadronization, below which parton dynamics is not valid. Because hadronization and then hadronic cascade are not yet included in BAMPS, a gluon, which ceases to interact, propagates freely and can be regarded as a free pion employing a picture of parton-hadron duality. Implementing a Cooper-Frye prescription for hadronization and employing UrQMD [16] for the hadronic cascade are in progress.

The minijet initial conditions and the subsequent evolution using the present prescription of BAMPS for two sets of the coupling $\alpha_s = 0.3$ and 0.6 give nice agreements to the measured transverse energy per rapidity over all rapidities [11, 17].

3. Thermalization of gluon matter

To study possible thermalization of gluons we concentrate on the local central region which is taken as an expanding cylinder with a radius of 1.5 fm and within an interval of space time rapidity $-0.2 < \eta < 0.2$, where $\eta = \frac{1}{2} \ln[(t+z)/(t-z)]$. Figure 1 shows the varying transverse momentum spectrum with time obtained from the BAMPS calculations for central Au+Au collisions at $\sqrt{s} = 200 \text{ A GeV}$, with elastic pQCD $gg \rightarrow gg$ only (left panel) and including pQCD-inspired bremsstrahlung $gg \leftrightarrow ggg$ (right panel), respectively. $\alpha_s = 0.3$ is used. One clearly sees that not much happens in the left panel of figure 1. With only elastic pQCD interactions the gluon system is initially nonthermal and also stays in a nonthermal state at late times. The situation

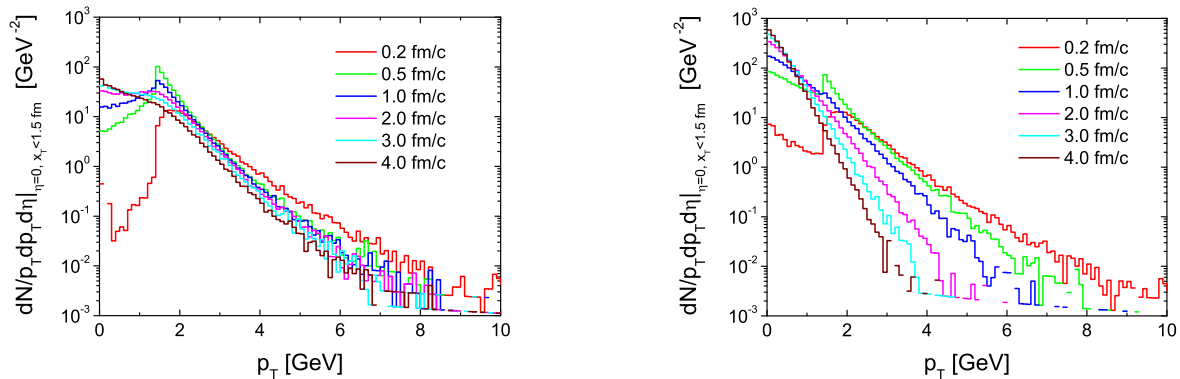


Figure 1. Transverse momentum spectrum in the central region at different times obtained from the BAMPS simulation with $gg \rightarrow gg$ only (left panel) and including $gg \leftrightarrow ggg$ collisions (right panel).

is dramatically changed when the inelastic interactions are included. In the right panel of figure 1 the curves from the upper to lowest depict, respectively, the spectrum at $t = 0.2, 0.5, 1, 2, 3$, and 4 fm/c. We see that the spectrum reaches an exponential shape at 1 fm/c and becomes increasingly steeper at late times. This is a clear indication for the achievement of local thermal equilibrium and the onset of hydrodynamical collective expansion with subsequent cooling by longitudinal work.

The inelastic pQCD-based bremsstrahlung and its back reaction are essential for the achievement of local thermal equilibrium at a short time scale. The fast thermalization happens in a similar way if color glass condensate is chosen as the initial conditions [19]. One of the important messages obtained there is that the hard gluons thermalize at the same time as the soft ones due to the $ggg \rightarrow gg$ process, which is not included in the “Bottom-Up” scenario of thermalization [18].

4. Significance of $gg \leftrightarrow ggg$ in thermalization and its crucial role in viscosity

To understand the efficiency of the pQCD $gg \leftrightarrow ggg$ processes for thermalization, we first calculate the collision rates $R_i = n \langle v_{\text{rel}} \sigma_i \rangle$, $i = gg \rightarrow gg, gg \rightarrow ggg$, using equations (1) and (2) in a thermal gluon gas. We find that $R_{gg \rightarrow gg}$ is larger than $R_{gg \rightarrow ggg}$, which implies that the collision rate is not the correct quantity determining the contribution of different processes to thermalization. Moreover, for instance, $\langle v_{\text{rel}} \sigma_{gg \rightarrow gg} \rangle = 0.82$ mb and $\langle v_{\text{rel}} \sigma_{gg \rightarrow ggg} \rangle = 0.57$ mb at temperature $T = 400$ GeV and $\alpha_s = 0.3$. A small value of cross sections can still lead to a fast equilibration.

Kinetic equilibration relates to momentum deflection. Large momentum deflections due to large-angle scatterings will speed up kinetic equilibration enormously. Whereas the elastic pQCD scatterings favor small-angle collisions, the collision angles in bremsstrahlung processes are almost isotropically distributed at RHIC energy due to the incorporation of the LPM effect [8, 9]. This is the intuitive reason why the bremsstrahlung processes are more effective in equilibration than the elastic interactions.

Quantitatively we demonstrated in [9] that the contributions of different processes to momentum isotropization are quantified by the so-called transport rates

$$R_i^{\text{tr}} = \frac{\int \frac{d^3p}{(2\pi)^3} \frac{p_z^2}{E^2} C_i - \langle \frac{p_z^2}{E^2} \rangle \int \frac{d^3p}{(2\pi)^3} C_i}{n \left(\frac{1}{3} - \langle \frac{p_z^2}{E^2} \rangle \right)}, \quad (3)$$

where $C_i[f]$, functional of the gluon density distribution $f(p, x)$, is the corresponding collision term describing various interactions, $i = gg \rightarrow gg, gg \rightarrow ggg, ggg \rightarrow gg$, respectively. Their sum gives the inverse of the time scale of momentum isotropization, which also marks the time scale of overall thermalization. To obtain R_i^{tr} close to thermal equilibrium we take $f = e^{-\beta E} \left(1 - \frac{\chi}{2} \beta \frac{p_z^2}{E} \right)$ with a small χ and calculate the term in zeroth order of χ in equation (3) [10]. Figure 2 shows the transport collision rate, scaled by temperature $T = 1/\beta$, for elastic $gg \rightarrow gg$ scattering and bremsstrahlung $gg \rightarrow ggg$, respectively. $R_{gg \rightarrow ggg}^{\text{tr}}$ is a factor of 3 – 5 larger than $R_{gg \rightarrow gg}^{\text{tr}}$ over a range in

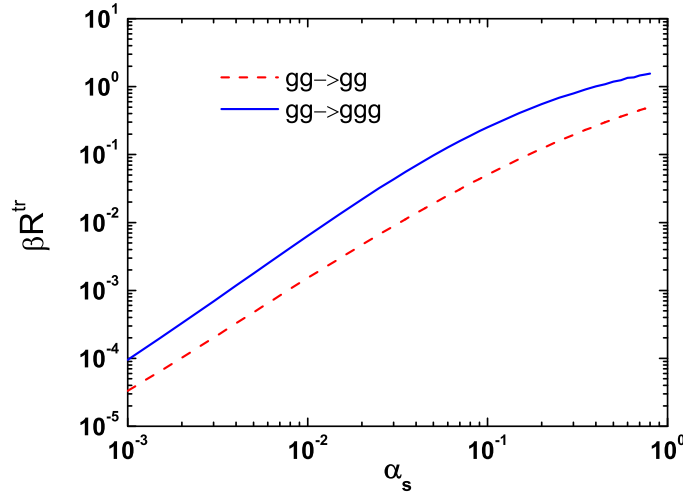


Figure 2. Transport collision rates scaled by temperature.

the coupling constant α_s from 10^{-3} to 0.8, which demonstrates the essential role of the bremsstrahlung in thermal equilibration. Because the transport rate (3) is expressed by characteristic moments of the collision term, it certainly contains a distribution of the collision angles, although it is not directly visible as in the transport cross section $\sigma_i^{\text{tr}} = \int d\sigma_i \sin^2 \theta$.

For a gluon gas, which is initially far away from equilibrium, we can roughly estimate the time scale of thermalization τ_{eq} by taking the inverse of the sum of the transport collision rates close to thermal equilibrium. At temperature $T = 400$ MeV we obtain $\tau_{\text{eq}} \approx 1/\sum R^{\text{tr}} = 0.32$ fm/c for $\alpha_s = 0.3$. We note that the above hinges on the assumption that the system is static. Expanding systems are more complicated because particles flow, which drives the systems out of local equilibrium. Therefore, the momentum degradation of flowing particles toward isotropy is slower than the inverse of the total transport collision rate [9].

Using the Navier-Stokes approximation we have derived the shear viscosity η from relativistic kinetic theory and found [10]

$$\eta \cong \frac{1}{5} n \frac{\langle E(\frac{1}{3} - \frac{p_z^2}{E^2}) \rangle}{\frac{1}{3} - \langle \frac{p_z^2}{E^2} \rangle} \frac{1}{\sum R^{\text{tr}} + \frac{3}{4} n \partial_t (\ln \lambda)}, \quad (4)$$

where λ denotes the gluon fugacity. This expression gives a direct correspondence of the macroscopic quantity η to its microscopic origin: η is inversely proportional to the sum of the total transport collision rate and the chemical equilibration rate, and it is roughly proportional to the energy density ϵ . At thermal equilibrium we obtain

$$\eta = \frac{4}{15} \frac{\epsilon}{\sum R^{\text{tr}}} \quad (5)$$

and the shear viscosity to entropy ratio is

$$\frac{\eta}{s} = \left(5\beta \sum R^{\text{tr}} \right)^{-1} = \left(5\beta R_{gg \rightarrow gg}^{\text{tr}} + \frac{25}{3} \beta R_{gg \rightarrow ggg}^{\text{tr}} \right)^{-1} \quad (6)$$

where the entropy density $s = \frac{4}{3} \beta \epsilon$ is used. Within the present description bremsstrahlung and its back reaction lower the shear viscosity to entropy density ratio significantly by a factor of 7, compared with the ratio when only elastic collisions are considered. For $\alpha_s = 0.3$ we obtain $\eta/s = 0.13$. To match the lower bound of $\eta/s = 1/4\pi$ from the AdS/CFT conjecture [7] $\alpha_s = 0.6$ has to be chosen. Even for that case the cross sections are in the order of 1 mb for a temperature of 400 MeV. We see that perturbative QCD interactions can drive the gluon matter to a strongly coupled system with an η/s ratio as small as the lower bound from the AdS/CFT conjecture.

5. Collective flow v_2 and extraction of shear viscosity

The elliptic flow $v_2 = \langle (p_x^2 - p_y^2)/p_T^2 \rangle$ can be directly calculated from microscopic simulations for Au+Au collisions at $\sqrt{s} = 200$ A GeV employing BAMPS. $\alpha_s = 0.3$ and 0.6 are used for comparisons. For a firm footing we compare our results with the experimental data, assuming parton-hadron duality. The left panel of figure 3 shows the elliptic flow v_2 at midrapidity for various centralities (impact parameters), compared with the PHOBOS [20] and STAR [21] data. Except for the central centrality region the results with $\alpha_s = 0.6$ agree perfectly with the experimental data, whereas the results with $\alpha_s = 0.3$ are roughly 20% smaller. We see that the generation of the large elliptic flow observed at RHIC is well described by pure perturbative gluon interactions as incorporated in BAMPS.

To see how viscous the gluon plasma behaves, the ratio of the shear viscosity to the entropy density is extracted by using equation (4) in those spatial regions where the matter is nearly equilibrated and, thus, behaves quasi-hydrodynamically, i.e., like a viscous fluid. The entropy density is estimated by $s = 4n - n \ln \lambda$ assuming that the gluon matter is in local kinetic equilibrium. The true entropy density is expected to be (slightly) smaller than that calculated by the above formula, because overall kinetic

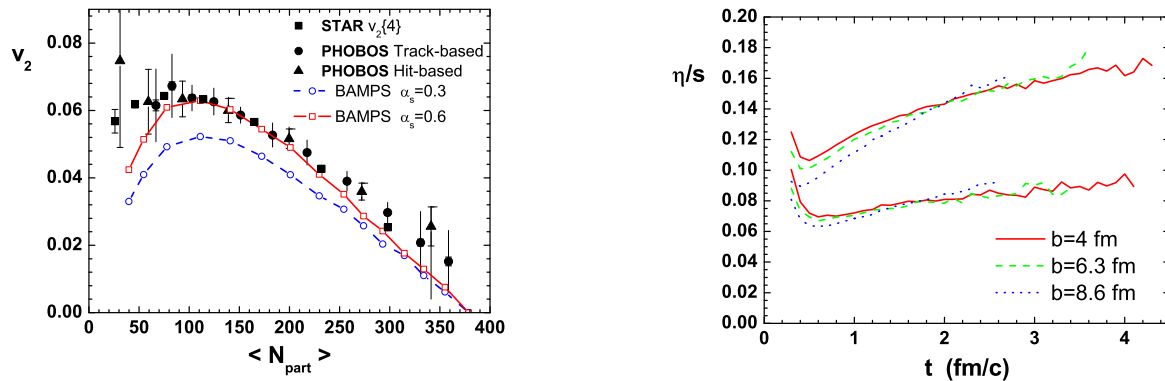


Figure 3. Left panel: elliptic flow $v_2(|y| < 1)$ from BAMPS using $\alpha_s = 0.3$ and 0.6 , compared with the PHOBOS [20] and STAR [21] data. Right panel: the shear viscosity to entropy density ratio η/s at the central region during the entire expansion. η/s values are extracted from the simulations at impact parameter $b = 4, 6.3$, and 8.6 fm. The upper band shows the results with $\alpha_s = 0.3$ and the lower band the results with $\alpha_s = 0.6$.

equilibration cannot be complete in an expanding system. Thus, the true shear viscosity to entropy density ratio is (slightly) larger than that calculated.

The ratio of the shear viscosity to the entropy density, η/s , is shown in the right panel of figure 3. We see that the ratio does not depend strongly on gluon density or temperature, since interaction rates and transport collision rates scale with the temperature. Hence, η/s depends practically only on α_s . For $\alpha_s = 0.6$, at which the v_2 values match the experimental data, we obtain $\eta/s \approx 0.08$, which is the same as the lower bound from the AdS/CFT conjecture. However, η/s may be higher, since the inclusion of hadronization [22] and subsequent hadronic cascade [23] will yield, even only moderate, contributions to the final elliptic flow values. Furthermore, different picture of initial conditions (e.g. color glass condensate) will also lead to different initially spatial eccentricity and, hence, will affect the final value of v_2 and the result on η/s . These investigations are underway and will provide more constraints on extracting η/s .

Note that the relation between η/s and v_2 (as a function of $\langle N_{\text{part}} \rangle$) is amazingly consistent with recent findings from a special set of viscous hydrodynamical calculations [6]. This certainly deserves further investigations.

6. Conclusion

The pQCD-based parton cascade BAMPS is used to calculate the time scale of thermalization, the elliptic flow v_2 and to extract the ratio of the shear viscosity to the entropy density, η/s , from simulations of Au+Au collisions at RHIC energy $\sqrt{s} = 200$ A GeV. This is a committed and large-scale undertaking. The present BAMPS includes elastic $gg \rightarrow gg$ and inelastic bremsstrahlung and its back reaction $gg \leftrightarrow ggg$. We observed that the gluon matter thermalizes at 1 fm/c for central collisions. Agreement

with the experimental data on v_2 is found with Glauber-type minijets initial conditions and $\alpha_s = 0.3 - 0.6$. The η/s ratio of the gluon plasma created varies between 0.08 and 0.15. Standard pQCD interactions alone can describe the generation of large v_2 values at RHIC. The small η/s ratios found in the simulations indicate that the gluon plasma created behaves like a nearly perfect fluid. This can be understood by perturbative QCD without resorting to exotic explanations such as the AdS/CFT conjecture. Gluon bremsstrahlung dominates and yields rapid thermalization, and, therefore, early pressure buildup, and a small shear viscosity in the gluon gas. Many further analyses on jet quenching and quark degrees of freedom including hadronization are underway to establish a more global picture of heavy-ion collisions.

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